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A Critical Path Problem Using Intuitionistic Triangular Fuzzy Number

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ABSTRACT
Critical path method is a network based method planned for development and organization of complex project in real world application. In this paper, a new methodology has been made to find the critical path in a directed acyclic graph, whose activity time uncertain. The vague parameters in the network are represented by intuitionistic triangular fuzzy numbers, instead of crisp numbers. A new procedure is proposed to find the optimal path, and finally illustrative examples is provided to validate the proposed approach.

Keywords **-** Critical path, Graded mean integration representation of intuitionistic triangular fuzzy number, Intuitionistic triangular fuzzy number, Order relation, Ranking of intuitionistic fuzzy number.

I. INTRODUCTION

A constructed network is an imperative tool in the development and organizes of definite project execution. Network diagram play a vital role in formative project-completion time. In general a project will consist of a number of performance and some performance can be started, only after ultimate some other behavior. There may be some behavior which is self-determining of others. Network analysis is a practice which determines the various sequences of behavior in relation to a project and the corresponding project completion time. The method is widely used are the critical path method program assessment and evaluation techniques. The successful finishing of critical path method requires the clear unwavering time duration in each movement. However in real life situation vagueness may arise from a number of possible sources like: due date may be distorted, capital may unavailable weather situation may root several impediments. Therefore the fuzzy set theory can play a significant role in this kind of problems to handle the ambiguity about the time duration of deeds in a project network. To effectively deal with the lack of clarity involved in the process of linguistic predictable times the intuitionistic trapezoidal fuzzy numbers are used to distinguish the fuzzy measures of linguistic values. In the current past, fuzzy critical path problems are addressed by many researchers, like S.Chanas , T.C.Han, J.Kamburowski, Zielinski, L.Sujatha and S.Elizabeth. S.Chanas, and J.Kamburowski,[11] explained fuzzy variables PERT. S.Chanas P.and Zielinski, [10] have also discussed critical path analysis in networks. G.Liang and T.C.Han [4] proposed a fuzzy critical path for project networks. Elizabeth and L.Sujatha [13] discussed a critical path problem under fuzzy Environment. They have deliberated a critical path problem for project networks. C.T Chen and S.F Huang [2] proposed a new model that combines fuzzy set theory with the PERT technique to determine the critical degrees of activities and paths, latest and earliest starting time and floats. The Bellman algorithm seeks to specify the critical path and the fuzzy earliest and latest starting time and floats of activities in a continuous fuzzy network. S.H. Nasution [12] proposed a fuzzy critical path method by considering interactive fuzzy subtraction and by observing that only the nonnegative part of the fuzzy numbers can have physical elucidation.

This paper is organized as follows: In section 2, basic definitions of intuitionistic fuzzy set theory have been reviewed. Section 3, give procedures to find out the intuitionistic fuzzy critical path using an illustrative example. In section 4 the obtained results are discussed. Finally, in section 5 some conclusions are drawn.

II. PRELIMINARIES

Some basic definitions related to our research work are review.

2.1 Intuitionistic Fuzzy Set

Let X be an Universe of discourse, then an Intuitionistic fuzzy set(IFS) A in X is given by $A=\left\{\left(x,\mu_A(x),\gamma_A(x)\right)/x\in X\right\}$, where the function

$\mu_A(x): X \to [0,1]$ and $\gamma_A(x): X \to [0,1]$ determine the degree of membership and non-membership of the element $x \in X$, respectively and for every $x \in X$, $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

2.2 Triangular Intuitionistic Fuzzy Number

An intuitionistic fuzzy number A={<a,b,c> <e,b,f>} is said to be a triangular intuitionistic fuzzy number if its membership function and non- membership function are given by

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \le x \le b \\ 1 & x = b \\ \frac{(c-x)}{(c-b)} & b \le x \le c \end{cases} \text{ where } a,b,c \in R \;\&$$

$$\gamma_A(x) = \begin{cases} \frac{(b-x)}{(b-e)} & e \le x \le b \\ 0 & x = b \\ \frac{(x-b)}{(f-b)} & b \le x \le f \end{cases} \text{ where } e,b,f \in R$$

2.3 Graded Mean Integration Representation for Triangular Intuitionistic Fuzzy numbers

The membership and non-membership function of triangular intuitionistic fuzzy numbers are defined as follows.

$$L_\mu(x) = \frac{x-a}{b-a} w \;; a \le x \le b \;\&\; R_\mu(x) = \frac{c-x}{c-b} w \;; b \le x \le c$$

$$L_\gamma(x) = \frac{b-x}{b-e} w \;; e \le x \le b \;\&\; R_\gamma(x) = \frac{x-b}{f-b} w \;; b \le x \le f$$

Then L^{-1} and R^{-1} are inverse functions of functions L and R respectively,

$$L_\mu^{-1}(h) = a + (b-a)\,h/w \;\&\; R_\mu^{-1}(h) = c - (c-b)\,h/w$$

$$L_\gamma^{-1}(h) = b - (b-e)\,h/w \;\&\; R_\gamma^{-1}(h) = b + (f-b)\,h/w$$

Then the graded mean integration representation of membership function and non-membership function are,

$$P_\mu(A) = \frac{a+4b+c}{6} \text{ - - - - - (1) \&}$$

$$P_\gamma(A) = \frac{b+e+f}{3} \text{ - - - - - (2)}$$

2.4 Arithmetic Operations of Triangular Intuitionistic Fuzzy Number

If $\tilde{A}^I = (< a_1, a_2, a_3 > ; < a'1, a_2, a'3 >)$ and $\tilde{B}^I = (< b_1, b_2, b_3 > ; < b'1, b_2, b'3 >)$

are two intuitionistic fuzzy numbers we define,

Addition :

$$\tilde{A}^I + \tilde{B}^I = (< a_1 + b_1, a_2 + b_2, a_3 + b_3 >; < a'1 + b'1, a_2 + b_2, a'3 + b'3 >)$$

Subtraction:

$$\tilde{A}^I - \tilde{B}^I = (< a_1 - b_3, a_2 - b_2, a_3 - b_1 >; < a'1 - b'3, a_2 - b_2, a'3 - b'1 >)$$

2.5 Order Relation

Consider an order relation among intuitionistic fuzzy numbers.Rule:Let A and B be two triangular intuitionistic fuzzy numbers such that $A = (< a_{\mu_1}, a_{\mu_2}, a_{\mu_3}; \mu_A >< c_{\gamma_1}, c_{\gamma_2}, c_{\gamma_3}; \gamma_A >)$ and

$B = (< b_{\mu_1}, b_{\mu_2}, b_{\mu_3}; \mu_A >< d_{\gamma_1}, d_{\gamma_2}, d_{\gamma_3}; \gamma_A >)$

with $\mu_A \ne \mu_B$ and $\gamma_A \ne \gamma_B$

$$1.\ a_{\mu_1} \le b_{\mu_1};\ c_{\gamma_1} \ge d_{\gamma_1} \quad 2.\ a_{\mu_2} \le b_{\mu 2};\ c_{\gamma_2} \ge d_{\gamma_2}$$
$$3.\ a_{\mu_3} \le b_{\mu_3};\ c_{\gamma_3} \ge d_{\gamma_3} -----(3)$$

2.6 Ranking of Intuitionstic Triangular Fuzzy umber

A Triangular intuitionistic fuzzy number

$$\tilde{A}' = \{(a, b, c); (e, b, f)\}$$

Then the graded mean integration representation of membership function and non-membership function of $\tilde{A}'$ are

$$P_{\mu}(\tilde{A}') = \frac{\alpha(a-c)+c+2b}{3} \quad \&$$

$$P_{\gamma}(\tilde{A}') = \frac{2\beta(e-f)+b+2f}{3}$$

Let $\tilde{A}' = (< a, b, c >< e, b, f >)$and
$\tilde{B}' = (< a', b, c' >< e', b, f' >)$ be any two
Intuitionistic triangular fuzzy numbers then

(i) $P_{\mu}^{\alpha}(\tilde{A}') < P_{\mu}^{\alpha}(\tilde{B}')$ and $P_{\gamma}^{\beta}(\tilde{A}') < P_{\gamma}^{\beta}(\tilde{B}')$

$\therefore\ \tilde{A}' < \tilde{B}'$.

(ii) ${}_{\mu}^{\alpha}(\tilde{A}') > P_{\mu}^{\alpha}(\tilde{B}')$ and $P_{\gamma}^{\beta}(\tilde{A}') > P_{\gamma}^{\beta}(\tilde{B}')$

$\therefore\ \tilde{A}' > \tilde{B}'$.

(iii)${}_{\mu}^{\alpha}(\tilde{A}') = P_{\mu}^{\alpha}(\tilde{B}')$ and $P_{\gamma}^{\beta}(\tilde{A}') P_{\gamma}^{\beta}(\tilde{B}')$

$\therefore\ \tilde{A}' \approx \tilde{B}'$.

III. INTUITIONISTIC FUZZY CRITICAL PATH METHOD

The following is the Procedure for finding intuitionistic fuzzy critical path.

3.1 Notations

N= The set of all nodes in a project network.
EST = Earliest Starting time
$EFT_{\mu ij}$ = Earliest finishing time for membership function
$EFT_{\gamma ij}$ = Earliest finishing time for non-membership function
$LFT_{\mu ij}$ = Latest finishing time for membership function
$LFT_{\gamma ij}$ = Latest finishing time for non- membership function
$LST_{\mu ij}$ = Latest starting time for membership function
$LST_{\gamma ij}$ = Latest starting time for non-membership function
TF = Total float
T_{ij} = The Intuitionistic fuzzy activity time

3.2 Forward Pass Calculation

Forward pass calculations are employed to calculate the Earliest Starting Time(EST) in the Project network

$$\bar{E}\mu\, j = \mathrm{Max}_{\,i} \left[\bar{E}_{\mu\, i}(+)\ \bar{t}\mu\ i\, j\right],$$

i = number of preceding nodes

$$\bar{E}\gamma\, j = \mathrm{Min}_{\,i} \left[\bar{E}_{\gamma\, i}(+)\ \bar{t}\gamma\ i\, j\right],$$

i = number of preceding nodes $-------(4)$

Earliest Finishing time for membership function

$$EFT_{\mu\, i\, j} = \left[EST_{\mu\, i\, j}(+)\ \text{Intuitionistic Fuzzy activitytime}\right]$$

Earliest Finishing time for non – membership function

$$EFT_{\gamma\, i\, j} = \left[EST_{\gamma\, i\, j}(+)\ \text{Intuitionistic Fuzzy activitytime}\right] ---(5)$$

3.3 Backward Pass Calculation

Backward pass calculations are employed to calculate the Latest Finishing Time (LFT) in the Project network

$$\bar{L}\mu\, i = Min_{\,j} \left[\bar{L}_{\mu\, j}(-)\ \bar{t}\mu\ i\ j\right],$$

j = number of succeeding nodes

$$\bar{L}\gamma\, j = Max_{\,i} \left[\bar{L}_{\gamma\, i}(-)\ \bar{t}\gamma\ i\ j\right],$$

j = number of succeeding nodes $-------(6)$

Latest Starting time for membership function

$$LST_{\mu\, i\, j} = \begin{bmatrix} LFT_{\mu\, i\, j}(-) \\ \text{Intuitionistic Fuzzy activitytime} \end{bmatrix}$$

Latest Finishing time for non – membership function

$$LST_{\gamma\, i\, j} = \begin{bmatrix} LFT_{\gamma\, i\, j}(-) \\ \text{Intuitionistic Fuzzy activitytime} \end{bmatrix} --(7)$$

3.4 Total Float (TF)

$$TF_{\mu} = LFT_{\mu ij} - EFT_{\mu ij} \text{ or}$$

$$TF_{\mu} = LST_{\mu ij} - EST_{\mu ij}$$

$$TF_{\gamma} = LFT_{\gamma ij} - EFT_{\gamma ij} \text{ or}$$

$$TF_{\gamma} = LST_{\gamma ij} - EST_{\gamma ij} \quad ----(8)$$

3.5 Procedure to Find Intuitionistic Fuzzy Critical Path

Step1: Construct a network G (V,E) where v is the set of vertices and E is the set of edges. Here G is an acyclic digraph and arc length or edge weight are taken as Intuitionistic Triangular fuzzy numbers.

Step 2: Expected time in terms of Intuitionistic Triangular fuzzy numbers are defuzzified using equation 1 and 2 in the network diagram.

Step3: Calculate Earliest Starting time for membership and non-membership functions ($EST_{\mu ij}$ and $EST_{\gamma ij}$ respectively) according to Forward pass calculation given in equation (4)

Step4: Calculate Earliest finishing time for membership and non-membership functions ($EFT_{\mu ij}$ and $EFT_{\gamma ij}$ respectively)using equation (5).

Step 5 : Calculate latest finishing time- $LFT_{\mu ij}$ and $LFT_{\gamma ij}$ - according to backward pass calculation given in equation(6)

Step 6: Calculate latest starting time -$LST_{\mu ij}$ and $LST_{\gamma ij-}$ using equation (7)

Step 7 : Calculate Total float- TF_{μ} and TF_{γ} -using equation (8)

Step 8: In each activity using (8) whenever one get 0 , such activities are called as Intuitionistic Fuzzy critical activities and the corresponding paths Intuitionistic critical paths.

3.5 Illustrative example

Consider a small network with 5 vertices and 6 edges shown in figure1, where each arc length is represented as a trapezoidal Intuitionistic fuzzy number

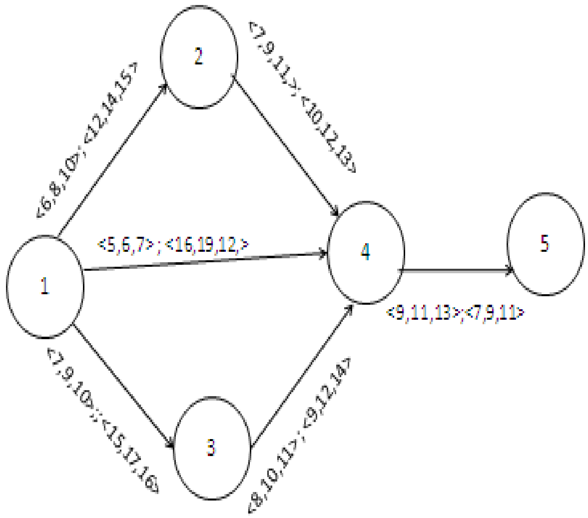

Fig 1

Table1: Results of the network

Activity	Intuitionistic Fuzzy Activity time	Defuzzified Activity time using equation 1 and 2 for membership and non-membership functions	TF_{μ}	TF_{γ}
1→2	<6,8,10> <12,14,15>	<8,13.7>	0	2.3
1→3	<7,9,10> <15,17,16>	<8.8,16>	12.6	0
1→4	<5,6,7> <16,19,12>	<6,15.7>	0	0
2→4	<7,9,11> <10,12,13>	<9,11.7>	11	2.3
3→4	<8,10,11> <9,12,14>	<9.8, 11.7>	12.6	0
4→5	<9,11,13> <7,9,11>	<11,9>	0	0

Table2: Rank value of total slack intuitionstic fuzzy time of all possible paths for membership function

Paths	IFCPM($p_{\mu k}$) k=1 to m	**Rank value definition 2.6**	**Rank**
1→2→4→5	**<22,28,34>**	**28**	**II**
1→4→5	<14,17,20>	**17**	**I**
1→3→4→5	<24,30,34>	**29.7**	**III**

Table3: Rank value of total slack intuitionstic fuzzy time of all possible paths for non-membership function

Paths	IFCPM($p_{\gamma k}$) k=1 to m	**Rank value definition 2.6**	**Rank**
1→2→4→5	**<29,35,39>**	**34.3**	**II**
1→4→5	<23,28,23>	**24.7**	**I**
1→3→4→5	<31,38,41>	**36.7**	**III**

IV. RESULTS AND DISCUSSION

This paper proposes an algorithm to tackle the problem in intuitionistic fuzzy environment. In this paper the trapezoidal intuitionistic fuzzy number is defuzzified using graded mean integration representation. Now the intuitionistic fuzzy number is converted to crisp number. Then applying the proposed algorithm we find the critical path. The path in intuitionistic fuzzy project network are 1→4→5,1→2→4→5and 1→3→4→5.The critical path for intuitionistic fuzzy network for both membership and non-membership function are 1→4→5. Hence the procedure developed in this paper form new methods to get critical path, in intuitionistic fuzzy environment.

V. CONCLUSION

A new analytical method for finding critical path in an intuitionistic fuzzy project network has been proposed. We have used new defuzzification formula for trapezoidal fuzzy number and applied to the float time for each activity in the intuitionistic fuzzy project network to find the critical path. In general intuitionistic fuzzy models are more effective in determining critical paths in real project networks. This paper, use the Graded mean integration representation the Procedure to find the optimal path in an intuitionistic fuzzy weighted graph having help decision makers to decide on the best possible critical path in intuitionistic fuzzy environments.

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